Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

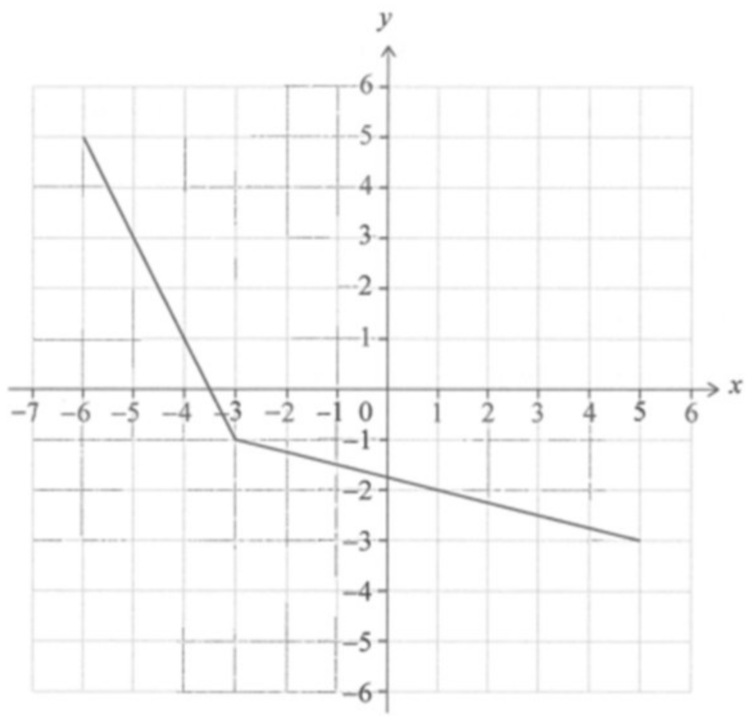
Let $\log_{10} 2 = p$ and $\log_{10} 3 = q$.

(a) Find an expression for $\log_{10} 24$ in terms of p and q. [3]

(b) Find an expression for $\log_3 8$ in terms of p and q. [2]

2. [Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$, for $-6 \le x \le 5$.



(a) Write down the value of $f(-3)$. [1]

(b) State the domain of f^{-1}, the inverse function of f. [1]

(c) Find the value of x that satisfies $f^{-1}(2x-7) = -3$. [3]

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3. [Maximum mark: 5]

Solve the equation $2\cos 2\theta - 5\cos\theta + 2 = 0$, where $\pi \le \theta \le 2\pi$.

4. [Maximum mark: 7]

Consider the curve $y = x^2 - x - 1$ and the line $y = mx - 3$, where $m \in \mathbb{R}$.

(a) Show that the curve and the line meet when $x^2 - (m+1)x + 2 = 0$. [2]

(b) Hence, find the values of m when the line is tangent to the curve. [5]

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5. [Maximum mark: 6]

The random variables X and Y are normally distributed with $X \sim \mathrm{N}(7, a^2)$ and $Y \sim \mathrm{N}(19, a^2)$, where $a > 0$.

(a) Find b such that $\mathrm{P}(X > b) = \mathrm{P}(Y > 22)$. [2]

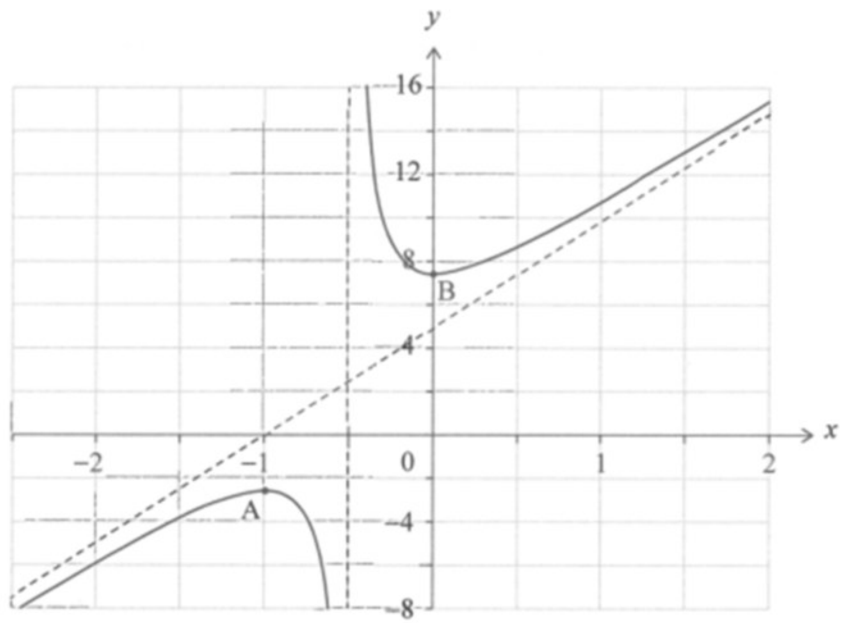
(b) Write down the approximate value of $\mathrm{P}(7 - a < X < 7 + a)$, correct to two significant figures. [1]

(c) Given that $a = 3$, calculate the approximate value of $\mathrm{P}(Y < 22)$, correct to two significant figures. [3]

6. [Maximum mark: 7]

Consider the function f. The graph of f has a local maximum at $\mathrm{A}\left(-1, -\frac{5}{2}\right)$, a local minimum at $\mathrm{B}\left(0, \frac{15}{2}\right)$, a vertical asymptote at $x = -\frac{1}{2}$ and an oblique asymptote $y = 5x + 5$.

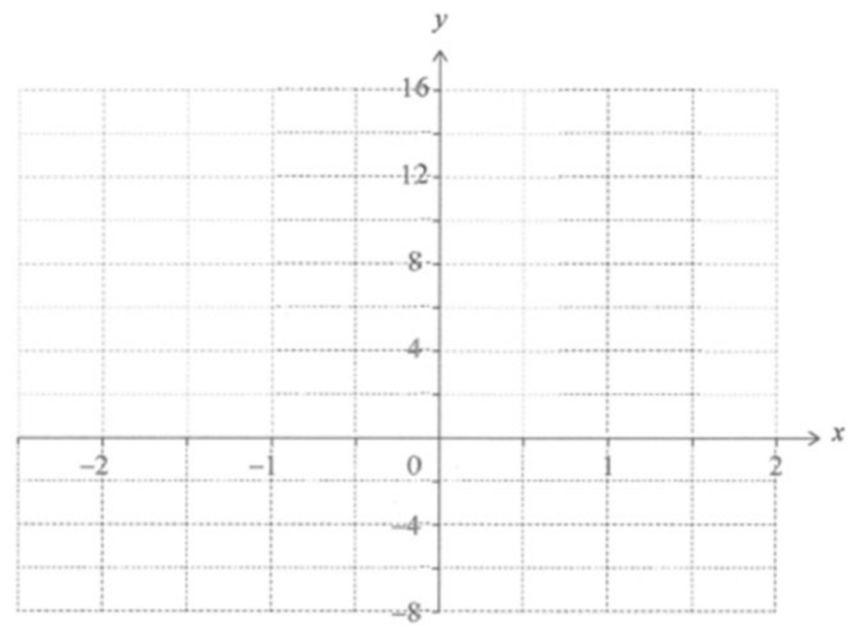
This information and part of the graph of f is shown in the following diagram.



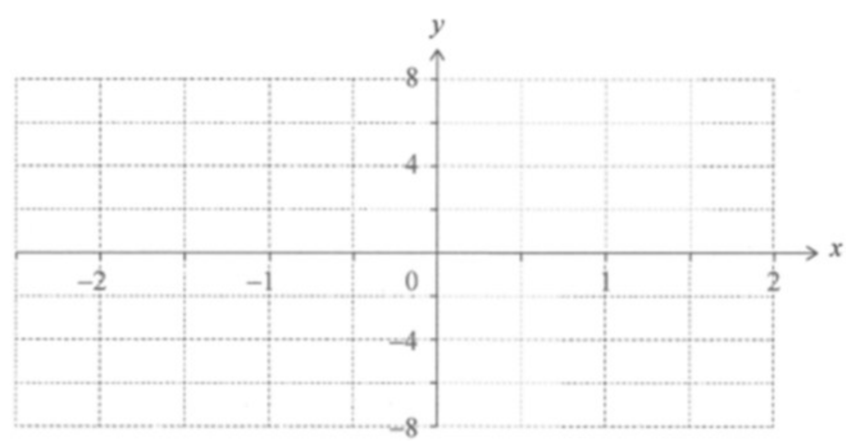
(This question continues on the following page)

(Question 6 continued)

(a) On the following grid, sketch the graph of $y = |f(x)|$, clearly indicating any asymptotes. [4]

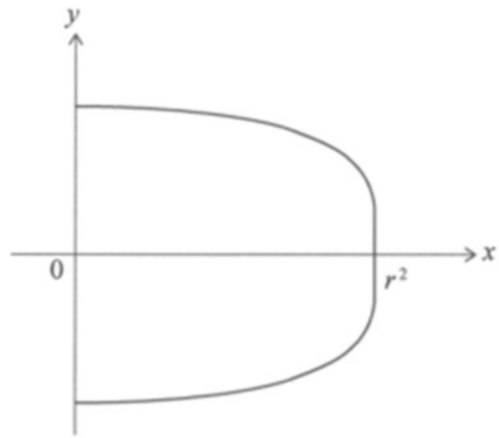


(b) On the following grid, sketch the graph of $y = \frac{15}{f(x)}$, clearly indicating any asymptotes and intercepts with the axes. [3]



7. [Maximum mark: 6]

The curve $x^2 + y^4 = r^4$, where $0 \le x \le r^2$, is shown in the following diagram.



The region enclosed by the curve and the y-axis is rotated through 2π radians about the y-axis to form a solid of revolution.

Find an expression for the volume of the solid in the form $V = a\pi r^b$, where a, $b \in \mathbb{Q}^+$.

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8. [Maximum mark: 8]

Consider the complex number $z_1 = \sqrt{3} - 3\mathrm{i}$.

(a) Express z_1 in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. [3]

Consider the complex number $z_2 = 2\sqrt{3}\mathrm{e}^{\mathrm{i}\frac{5\pi}{6}}$.

The cube roots of $\frac{z_2}{z_1}$ are denoted by u, v and w.

(b) Find u, v and w. [5]

9. [Maximum mark: 6]

Determine the value of $\lim_{x \to 0}\left(\frac{x\sin x}{1-\cos x}\right)$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

Consider the sequence $\{u_n\}$, with nth term given by u_n. The first three terms are

$$u_1 = k - 5,\ u_2 = 3 - 2k \text{ and } u_3 = 5k + 3, \text{ where } k \in \mathbb{R}.$$

(a) Consider the case when $\{u_n\}$ is arithmetic.

(i) Find the value of k.

(ii) Hence, or otherwise, find u_3. [5]

(b) Consider the case where $k = 12$.

(i) Show that the first three terms of $\{u_n\}$ form a geometric sequence.

(ii) Given that $\{u_n\}$ is geometric, state a reason why the sum of an infinite number of terms of this sequence does not exist. [4]

(c) The sequence, $\{u_n\}$, is geometric for a second value of k.

(i) Show that $k^2 - 10k - 24 = 0$.

(ii) Find the first three terms of $\{u_n\}$ for this second value of k.

(iii) Hence, write down the value of S_{2m}, the sum of the first $2m$ terms, for this second value of k. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 18]

The points $A(1, -4, 0)$, $B(-3, -6, 2)$, $C(-1, -2, 4)$ and D form a parallelogram, $ABCD$, where D is diagonally opposite B.

(a) Find the coordinates of D. [2]

The diagonals of the parallelogram, $[AC]$ and $[BD]$, intersect at point E.

(b) Find the coordinates of E. [2]

(c) (i) Given that $\vec{AB} \times \vec{AD} = m\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$, where $m \in \mathbb{Z}^+$, find the value of m.

(ii) Hence, find the area of parallelogram $ABCD$. [4]

The plane, Π_1, contains the parallelogram $ABCD$.

(d) Find the Cartesian equation of Π_1. [2]

A second plane, Π_2, has Cartesian equation $5x + y - 7z = 1$.

The acute angle between Π_1 and Π_2 is θ.

(e) Show that $\cos\theta = \frac{1}{5}$. [3]

The line L passes through E and is perpendicular to Π_1.

The line L intersects the plane Π_2 at point F.

(f) Find the coordinates of F. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

Consider the complex number $z = x + y\mathrm{i}$, where $x, y \in \mathbb{R}$, such that $|z - (2 + \mathrm{i})| = 3$.

(a) Show that $x^2 + y^2 - 4x - 2y - 4 = 0$. [3]

The argument of $\frac{z+p}{z-1}$ is $\frac{\pi}{4}$, where $p \in \mathbb{R}$.

(b) Show that $x^2 + y^2 + (p - 1)x + (p + 1)y - p = 0$. [7]

Two roots of the equation $z^4 + az^3 + bz^2 + cz + d = 0$ are z_1 and z_2, where $z \in \mathbb{C}$ and $a, b, c, d \in \mathbb{R}$.

Both z_1 and z_2 satisfy the conditions $|z - (2 + \mathrm{i})| = 3$ and $\arg\left(\frac{z+4}{z-1}\right) = \frac{\pi}{4}$.

(c) Use the results from parts (a) and (b) to find z_1 and z_2. [7]

(d) Find the value of a. [4]

Please **do not** write on this page.

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